Generalized Modified Group Chain Sampling Plan based on Non-symmetrical Data

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***Abstract*— Acceptance sampling is a quality control technique widely used by companies in deciding whether or not to accept a lot or batch of products based on the inspection result of a sample taken from the lot. Majority of acceptance sampling plans are developed assuming symmetrical data distribution, whereas actual scenarios include product lifetime which is often non-symmetrical. In this article, a generalized modified group chain sampling plan (GMGChSP) based on non-symmetrical data is proposed using preceding lot information. An iterative technique is employed to find the design parameters of the proposed plan by satisfying the pre-specified consumer's risks, number of testers and proportion defective. The performance of the proposed plan is compared to an established plan developed by Mughal et al. (2016b).**

Keywords*—Acceptance Sampling, Group sampling, modified chain sampling, consumer's risk.*

1. Introduction

In many industries, acceptance sampling is commonly used as a quality control measure of incoming materials, work-in progress or final products. Quality control department applies suitable acceptance sampling plans to inspect the product against a desired quality standard. These plans often assume that proportion defectives of submitted products are constant. However in real situations, there exist various uncontrolled factors such as environmental and material variability, which cause the proportion defective to vary from sample to sample.

In majority acceptance sampling plans for a truncated life test, the most important concern is to determine the sample size from the lot under examination. It is implicitly assumed in the ordinary acceptance sampling plans that only a single item is put in tester for inspection but in practical life testing, testers can accommodate more than one product at a time. The truncated life test based on this kind of testing is called group acceptance sampling.

Many researchers suggested various single and group acceptance sampling plans for various distributions: [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16] and [17]. For situations in which testing is costly or destructive, acceptance sampling plans with small sample sizes are generally preferred. This small sample size acceptance sampling plan often contains acceptance number c zero; this means that the entire lot can only be accepted if the sample inspected contains no defective unit. Any ordinary plan having zero acceptance numbers are often undesirable since the probability of lot acceptance begins to decrease very sharply for very small value of proportion defectives; this is deemed unfair to the producer due to inflated producer's risk of good lots being rejected. To overcome this drawback, Dodge [18] introduced a procedure known as chain sampling plan (ChSP-1) in 1955, and further development was accomplished by several researchers [2], [12], [20] and [19]. These plans offer favourable alternatives to the ordinary acceptance sampling plans.

As the name suggests, chain sampling plan considers the stream of lots as chain and utilizes the cumulative information of the preceding lots in deciding the disposition of the current lot. In other words, the criteria for accepting and rejecting the submitted lot depend on the inspection results of the immediately preceding samples. In 1998, Govindaraju and Lai [2] introduced modified chain sampling plan based on truncated life test. The preceding or prior information of the submitted lot

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was considered and the formula of operating characteristic function was derived from Poisson distribution. Comparative studies showed that the modified chain sampling plan is an improvement of the original chain sampling. The former is able to minimize consumer's risk of accepting bad lot while maintaining acceptable producer's risk.

Ramaswamy and Jayasri [12] developed chain sampling plan based on truncated lifetime that assumes a generalized Rayleigh distribution. Minimum sample size and the required acceptance number are obtained when satisfying the other pre-assumed design parameters at various quality levels. Ramaswamy and Jayasri [20] introduced modified chain acceptance sampling plan considering several lifetime distributions. For pre-specified values of test termination time and other design parameters, minimum sample sizes and operating characteristic values are obtained and discussed with the aid of numerical examples.

In ordinary acceptance sampling plans, only a single product is inspected at one time, but in practice, it is possible to inspect more than one product at the same time given the availability of testers. In this situation, the submitted products put in a tester are considered as group (multiple testers each accommodating g products) and such plan based on this type of inspection is known as group acceptance sampling plan.

Mughal and Aslam [8] introduced efficient group acceptance sampling plan for family Pareto distribution and total number of defectives products assumed as groups. The advantage of their proposed plan over existing plan developed by [5] was also elaborated. As mentioned earlier in life testing, a truncated life test is conducted to find the smallest sample size to ensure a specified mean life of a submitted product.

The established chain sampling plans were constructed based on mean lifetime for assessing the quality and reliability of products. However, the sampling plans based on the population mean may not be very useful when the lifetime of a product follows a specific percentile. Mughal et al. [7] developed an economic reliability acceptance sampling plan for Burr type XII distribution when the lifetime of product follow the non-symmetrical distribution theory.

Meanwhile, Rao [21] introduced acceptance sampling plan from truncated life test based on the Marshall-Olkin extended exponential distribution for percentile, when the life test is truncated at a pre-specified time. The operating characteristic (OC) value and OC curve of the sampling plan to determine producer's risk were discussed. The minimum sample size necessary to ensure the specified life percentile was obtained for a given customer's risk.

Rao et al. [22] proposed a group acceptance sampling plan developed based on truncated lifetimes when the lifetime of an item follows a half normal distribution. For a given group size, the minimum number of groups and the acceptance number required were determined for specified consumer's risk and the test termination time. The values of operating characteristic function for various quality levels were computed and the minimum ratios of the true average life to the specified life at given producer's risk were obtained. More recently, Mughal et al. [19] introduced time truncated group chain sampling strategy for Pareto distribution of the 2nd kind.

In this article, generalized modified group chain sampling plan (GMGChSP) is introduced by taking advantage of the cumulative information of preceding i, lots. Our proposed plans for truncated life test will be very useful to overcome the difficulties of handling the non-symmetrical data. The main focus of this plan is to find the optimal sample sizes and operating characteristic values by assuming the various values of design parameters.

2. Index of Symbols

- g : Number of groups.
- r : Group size.
- n : Sample size.
- d : Number of defective products.
- I : Allowable acceptance number (preceding lot)
- α : Producer's risk (Probability of rejecting a good lot).
- β : Consumer's risk (Probability of accepting a bad lot).
- $L(p)$: Lot acceptance probability.

3. Methodology

The proposed plan is functional in the following steps and also shown in Figure 1:

Step 1 Find the minimum number of g groups and allocate r products to each group such that the required sample size is $n=r\times g$.

Step 2 Inspect the sample and count the number of defectives, d.

Step 3 If no defective is found in the current sample ($d=0$) defectives, ($d\_i=0$), accept the lot.

Step 4 If no defective is found in the current sample ($d=0$), while the preceding i samples have only one defective ($d\_i=1$), accept the lot.

Step 5 If one or more defectives are found in the current sample ($d>0$), reject the lot.

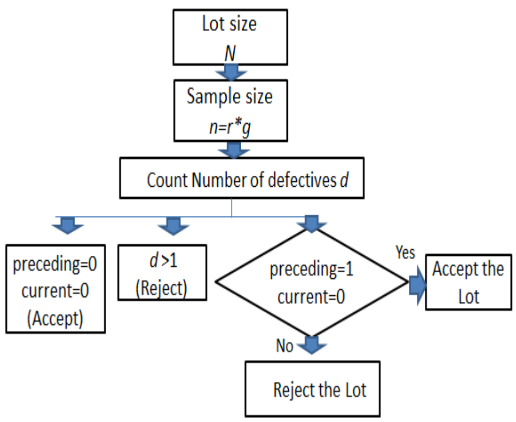

Figure 1. Acceptance sampling procedure for GMGChSP

The GMGChSP is characterized by the design parameters g and $L(p)$. Our prime interest is in obtaining the minimum group size and probability of lot acceptance at pre-specified values of β = 0.25, 0.10, 0.05, 0.01; r=2(1)5 and i=1(1)4.

Based on the above steps, the probability of the sample containing zero and one defective product for GMGChSP can be written in the following form by using the probability law of addition:

$$L(p)_{\text{GMGChSP}} = \{P(d=0)|P(d=0)_i\} + \{P(d=0)|P(d=1)_i\} \quad (1)$$

$$L(p)_{\text{GMGChSP}} = \left\{\left(P_{0,(r*g)}\right)^3 + 2P_{1,(r*g)}\left(P_{0,(r*g)}\right)^2\right\} \quad (2)$$

Following the above equation (2), the general expression of probability of lot acceptance for GMGChSP can be written as:

$$L(p)_{\text{GMGChSP}} = \left\{\left(P_{0,(r*g)}\right)^{(i+1)} + i\,P_{1,(r*g)}\left(P_{0,(r*g)}\right)^{i}\right\} \quad (3)$$

Considering Binomial distribution, the above equation (3) converts to the following forms:

$$L(p)_{\text{GMGChSP}} = \left[\left\{\binom{r*g}{0} p^0(1-p)^{r*g}\right\}^{(i+1)} + i\left\{\binom{r*g}{1} p^1(1-p)^{(r*g)-1}\right\}\left\{\binom{r*g}{0} p^0(1-p)^{r*g}\right\}^{i}\right] \quad (4)$$

Upon simplification of the above equation (4), the probability of lot acceptance of group chain GMGChSP becomes:

$$L(p)_{\text{GMGChSP}} = (1-p)^{(r*g)(i+1)}[1 + i(r*g)(p)/(1-p)] \quad (5)$$

where p denotes the proportion defectives. For various values of β, i, and r, the minimum number of groups, g, are presented in Table 1 using the above equation (5).

Table 1. Number of minimum groups required for GMGChSP for various values of proportion defective

			p					
β	r	i	0.10	0.15	0.20	0.25	0.30	0.35
0.25	2	1	6	4	3	2	2	1
	3	2	3	2	2	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	8	6	4	3	3	2
	3	2	4	3	2	2	2	1
	4	3	3	2	2	1	1	1
	5	4	2	1	1	1	1	1
0.05	2	1	10	7	5	4	3	2
	3	2	5	4	3	2	2	2
	4	3	3	2	2	2	1	1
	5	4	2	2	1	1	1	1
0.01	2	1	15	10	7	6	5	3
	3	2	7	5	4	3	2	2
	4	3	4	3	2	2	2	1
	5	4	3	2	2	1	1	1

As depicted in Table 1, the number of groups required for the GMGChSP varies for various values of consumer's risk but it decreases when the number of preceding lots, number of testers and pre-specified proportion defective increases. Considering consumer's risk, β=0.01, p=0.10, r=2, i=1, the required number of groups are 15. Meanwhile, at p=0.10, r=3, i=2, the required number of groups is 7. This implies that when the number of preceding lots and number of testers increase, a small number of groups is required to reach the valid conclusion about the submitted lot. The outcome of proportion defective and consumer's risk to the number of groups is presented in Table 2. The choices of design parameter values are considered only for comparison purposes.

Table 2. Number of minimum groups for r=3 and i=2

β	g	p	β	g	p	β	g	p
0.10	4	0.10	0.05	5	0.10	0.01	7	0.10
	3	0.15		4	0.15		5	0.15
	2	0.20		3	0.20		4	0.20
	2	0.25		2	0.25		3	0.25
	2	0.30		2	0.30		2	0.30
	1	0.35		2	0.35		2	0.35

In Table 2, the number of groups, g decreases when the proportion defective increases, but it increases when the consumer's risk decreases for a specified value of proportion defective. Consider, β=0.01, r=3, and i=2, the required number of groups is 7 when p=0.10. The number of groups decreases from 7 to 2 when proportion defective increases from p=0.10 to p=0.35. For a fixed proportion defective p=0.10, the number of groups decreases from 7 to 4 when consumer's risk increases from 0.01 to 0.10 respectively.

4. Comparisons

In this section, a comparison is made between the proposed plan and the established plan developed by Mughal et al. [23] for specified design parameters as displayed in Table 3.

Table 3. Comparisons of number of groups for r=3 and i=2.

β	p	Mughal *et al.*, (2016b) g	Proposed GMGChSP g
0.01	0.10	15	7
	0.15	10	5
	0.20	7	4
	0.25	6	3
	0.30	5	2
	0.35	4	2

Table 3 clearly shows that the GMGChSP requires smaller number of groups, and hence smaller sample size than the established plan.

5. Conclusion

The generalized modified group chain sampling plan (GMGChSP) developed in this paper is an improvement of the Mughal et al. [23] plan. The formula for the OC function of GMGChSP is derived and several tables based on the Binomial model are presented. Comparisons are provided to show that GMGChSP would require a smaller sample size over the Mughal et al. [23] plan. Such smaller sample size may lead to lower cost of performing the inspection as well as reduced risk of product damage due to mishandling. This evidence suggests that GMGChSP plan is likely to be suitable for assessments involving destructive or costly truncated testing by attributes. Among suitable applications are in

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