# Stochastic Inventory Control Systems with Consideration for the Cost Factors Based on EBIT 

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#### Abstract

Due to a short market life-span and high uncertainty in future demand, supply chain management is a competitive advantage which plays an important role in todays global semiconductor industry. A very important consequence of uncertain demand and having long lead time is the great risk of incurring shortages and excessive inventory. This paper considers the view of the second tier semiconductor supplier in automotive industries and studies, using the periodic review analysis, a single item single stage inventory system with stochastic demand. The only uncertainty is associated with demand. Assuming holding, production, salvage and backorder costs, we determine the optimal numerical value of the level s (reorder point) using a simulation approach, and thus obtain the optimal inventory policy to minimize the total expected inventory cost while being able to achieve the desired customer service levels.


Key words: Second Tier Supplier, Backorder, (s, Q) inventory model, Stochastic Demand, EBIT

## 1. Introduction

For the semiconductor industry, due to lack of visibility across the supply chain, minor disturbances in end demand can translate into huge disturbances at downstream suppliers (semiconductor industry's position). Distorted information, or what is called the Bullwhip Effect, can cause great inefficiencies which are excessive inventory investment, poor customer service, lost revenues, ineffective transportation, and missed production schedules [2].

[^0]Given this backdrop, it can significantly impact company's profit. With production taking place early in the value chain, lead-times are long, and the industry suffers from a heavy bullwhip effect [1]. This effect causes the forecast error. In the semiconductor industry, however, improved forecasting can only take you so far. A better forecast can save your company money by reducing inventory carrying costs and obsolescence. But because of the boom and bust nature of this industry, semiconductor companies also need extremely flexible supply chains. Moreover, in the current world the semiconductor industry is more competitive so companies are forced to maintain a high service level, to avoid being charged high backlog costs [8]. For the purpose of improving customer satisfaction and reducing inventories in the semiconductor supply chain and considering the impact of low forecasting accuracy and the unusual high backlog costs. One of the powerful tool to control the supply chain is Kanban system [5, 7, 9]. However, this tool is assumed to a smoothing of production.

This paper proposes a simple periodic review policy where no order is placed as long as the inventory position, defined as the stock on-hand plus stock on-order minus backorders, is equal or larger than the level $s$. Otherwise an order is placed as a fixed order quantity. The remainder of the paper is organized as follows. In section 2, the proposed inventory model is introduced; afterwards we describe setting safety stock based on desired service level. In section 3, cost factors which are related to the inventory model are described. Section 4 shows how
we define the order quantity in the model. In section 5 , all the notations are describes then the inventory simulation model is presented to test the performance of the policy. Section 6 concludes the paper with a summary.

## 2. Model description

There is no question that uncertainty plays a role in most inventory management situations [4]. The companies need enough supply to satisfy customer demands, but ordering too much increases holding costs and the risk of losses through obsolescence. An order too small increases the risk of lost sales and unsatisfied customers. We consider a single item single echelon system with stochastic demand. In order to manage the inventory and place replenishment orders, a periodic review system is used. We consider the ( $s, Q$ ) inventory policy, alternatively called the reorder point, order quantity system. This policy operates as follows: the inventory position is monitored. If the inventory position is higher than level s , then no order is triggered. In case the inventory position is below the level s , an amount is ordered which equals $Q$. The order arrives to replenish the inventory after a lead time, $L T$. The values of $s$ and $Q$ are the two decisions required to implement the policy. The lead time is assumed known and constant. The only uncertainty is associated with demand. Whenever demand cannot be satisfied directly from stock, demand is backordered. Backordered are satisfied when the next replenishment arrives. We may also be interested in the expected number of items backordered during an order cycle (see Figure 1 for an illustration of the policy).


Figure 1: The ( $s, Q$ ) policy ( $L T=16$ weeks)

The random demand during the lead time gives rise to the possibility that the inventory level will be depleted before the replenishment arrives. With the average rate of demand equal to a, the mean demand during the lead time is:

$$
\mu=a * L T
$$

A shortage will occur if the demand during the period L is greater that s .
Product availability is a key element of customer service for supply chain managers. One well-known customer service metric is the service level, which is the fraction of replenishment cycles in which demand is fully satisfied. The service level tells us whether a stock out event occurred during a replenishment cycle, but does not capture the quantity either backordered [10]. From a long-run service perspective, safety inventory is the average amount of net stock on hand kept as a buffer against demand and supply uncertainty. The difference between the amount of inventory available when placing a replenishment order and the expected demand during the stock out exposure period provides a close approximation of the safety inventory when service targets are high enough to make the expected units short per replenishment cycle inconsequential. A target service level for a single product represents the desired fraction of demand that is filled from available inventory.

Service level $=1-($ Backlog $/$ Demand $) \quad(1)$

The service level is the probability that the amount of inventory on hand during the lead time is sufficient to meet expected demand, that is, the probability that a stock out will not occur. In practical instances, the reorder point is significantly greater than the mean demand during the lead time so the safety stock (SS) is defined as:

$$
\begin{equation*}
\mathrm{SS}=\mathrm{s}-\mu \tag{2}
\end{equation*}
$$

A shortage will occur if the demand during the period L is greater that s. This probability is defined in Figure 2.


Figure 2: Distribution of Demand over Exposure Periods

The uncertainty in demand exposes company to out-of-stock risks for a particular period, which is the lead time ( $L T$ ). We must model the distribution of demand over this exposure period to determine safety inventory levels. The histogram in Figure 2 represents one possible distribution of demand over the exposure
period represented by the random variable $X$. The function $F(x)$ represents the cumulative distribution function ( $c d f$ ), which enables us to determine the probability that uncertain values of $X$ will be less than or equal to a particular value, such as the reorder point. This probability, defined as Ps , is calculated as:

$$
\begin{equation*}
\mathrm{Ps}=\mathrm{P}\{\mathrm{x}>\mathrm{s}\}=\int \mathrm{g}(\mathrm{x}) d x=1-\mathrm{G}(\mathrm{~s}) \tag{3}
\end{equation*}
$$

To compute the reorder point with a safety stock that will meet a specific service level, we will assume that the demand during the lead time is uncertain, independent, and can be described by a normal distribution. The reorder point(s) to meet a specific service level can be computed as:

$$
\begin{equation*}
\mathrm{s}=\mathrm{a}^{*} \mathrm{LT}+\mathrm{z}^{-1}(\mathrm{SL}) \theta(\mathrm{LT})^{1 / 2} \tag{4}
\end{equation*}
$$

Once the wished service level defined, this percentage will be used to get the corresponding " z -values", a statistical z table. The term in this formula for the reorder point is the square root of the sum of the daily variances during the lead time period. From equation (2) and equation (4), the reorder point relative to the service level is shown in equation (5) as:

$$
\begin{equation*}
\mathrm{SS}=\mathrm{z}^{-1}(\mathrm{SL}) \vartheta(\mathrm{LT})^{1 / 2} \tag{5}
\end{equation*}
$$

## 3. Cost factors based on EBIT

In order to evaluate the inventory system, the average costs per review period are considered, which are composed of two main components [3]. On the one hand the company incurs inventory holding costs and on the other hand backorder costs arise from stock outs. An inventory holding cost is charged for each unit in stock at the end of a period and a penalty cost is charged for each unit short at the end of a period.


Figure 3: Cost factor Trade Off in Inventory

There are 4 types of cost factors that are related to analyzing inventory problems. The four types are (1) ordering costs, (2) setup costs, (3) holding costs, and (4) backlog costs. It is necessary to examine the effects of low forecast accuracy and long lead time on total
profit for supply chain to minimize total inventory cost. The ordering cost is simply the total of expenses incurred in placing an order. The costs increase as the number of orders placed increases. They include costs related to setup costs. The ordering cost can be calculated as equation (6); note that $K$ is setup cost for placing an order, c is ordering cost per-unit and $\mathrm{Q}(\mathrm{t})$ is amount of placed orders:

$$
\begin{equation*}
\mathrm{K}+\mathrm{c}^{*} \mathrm{Q}(\mathrm{t}) ; \mathrm{Q}(\mathrm{t})>0 \tag{6}
\end{equation*}
$$

The holding costs encompass all the costs associated with holding the goods in inventory and its dimension is per unit per time. In the model, holding cost will be calculated weekly since the inventory on hand is updated weekly. The holding cost is shown in equation (7), where h is holding cost's dimension is given perperiod and per-unit of inventory, and $\mathrm{J}(\mathrm{t})$ is inventory on hand level.

$$
\begin{equation*}
\mathrm{h} * \max (\mathrm{~J}(\mathrm{t}), 0) \tag{7}
\end{equation*}
$$

The backlog costs are more difficult to quantify. The main component is lost future profit from lost future sales caused by customer dissatisfaction with delays in filling. This means that when the inventory is empty and additional demand occurs, customers will wait for delivery until the next inventory replenishment. During that time, a charge is incurred proportional to the time the customer must wait until delivery. In this paper, the backlog cost is considered in relation to EBIT (Earnings Before Interest Taxes) and service level. This relation can be expressed by a Quasi Exponential function. EBIT can be simply calculated by the subtraction of operating expense from revenue. The backlog cost can be calculated as following, where $b$ is backlog cost per units per time:

$$
\mathrm{b}(\mathrm{SL}) * \max (-\mathrm{J}(\mathrm{t}), 0)
$$



Figure 4: Quasi-Exponential function of Backlog cost per unit and Service Level
Table 1: The Cost Factors for Inventory Model

| The Cost Factors for Inventory Model |  |
| :--- | :--- |
| Revenue/unit, p | 100 |
| Ordering cost/unit, c | 0.6 p |
| Administrative cost (fixed), K | 0.25 p |
| EBIT/unit | $\mathrm{p}-(\mathrm{c}+\mathrm{K})$ |
| Holding cost/unit/week, h | 0.18 p |

Table 1 summarizes all these cost factors and their estimated values. Total Inventory cost is the total cost associated with holding cost, ordering cost and backlog cost, shown as following:

## Total cost

$$
\begin{align*}
& =\text { Holding cost }+ \text { Backlog cost }+ \text { Ordering cost } \\
& =h^{*} \max (\mathrm{~J}(\mathrm{t}), 0)+\mathrm{b}(\mathrm{SL})^{*} \max (-\mathrm{J}(\mathrm{t}), 0) \\
& +\left(\mathrm{K}+\mathrm{c}^{*} \mathrm{O}(\mathrm{t})\right) \tag{9}
\end{align*}
$$

## 4. Determining the order quantities

### 4.1 Forecast accuracy

Forecast accuracy at the primitive stocking unit level is critical for proper allocation of supply chain resources. In the semiconductor industry, many critical decisions are based on demand forecasts [6]. As described in the previous section, presence of long lead time and frequently changing demand signals result in low forecast accuracy. The model also takes into account a low forecast accuracy of $70 \%$ which is the actual forecast accuracy as a prediction in the production system. If there is a $70 \%$ chance that the forecasting is equal to actual demand, then the error score is based on 0.3. This is described in equation (9). Figure 5 shows the definition of $70 \%$ forecast accuracy:

$$
\begin{equation*}
\operatorname{Pr}(\mathrm{F}(\mathrm{t})=\mathrm{D}(\mathrm{t}))=0.7 \tag{10}
\end{equation*}
$$



Figure5 : Concept of $70 \%$ forecast accuracy model

We use the general economic order quantity (EOQ) model form to indicate the optimal value of Q which is given in the following equation,

$$
\begin{equation*}
\mathrm{Q}=[(\mathrm{h}+\mathrm{b}) / \mathrm{b}]^{1 / 2} *[(1 * \mathrm{~K} * \mathrm{a}) / \mathrm{h}]^{1 / 2} \tag{11}
\end{equation*}
$$

### 4.2 Order quantity

Backlog cost in this model is not a fixed value, it is a function of service level. Therefore, using the EOQ model to calculate the order quantity for this model might not be
appropriate. Therefore, we assumed order quantity, $\mathrm{Q}(\mathrm{t})$ as following equation:

$$
\begin{equation*}
\mathrm{Q}(\mathrm{t})=\mathrm{F}(\mathrm{t}) * \mathrm{LT} \tag{12}
\end{equation*}
$$

Considering the amount of order quantities should be able to cover the demand over the period of time until the next ordering will be placed. Based on the earlier mention in forecast accuracy section, the model is set under the scenario of having $70 \%$ forecast accuracy. This assumption is shown by the following equation.

$$
\begin{equation*}
\operatorname{Pr}(\mathrm{Q}=\mathrm{D}(\mathrm{t}))=0.7 \tag{13}
\end{equation*}
$$

Equation (13) leads to how this inventory model would have performed throughout the $70 \%$ forecast accuracy measure. Under this condition, we should determine the optimal safety stock for ordering the inventory quantity that minimizes the expected cost.

## 5. Numerical experiments

### 5.1 Notifications

We implement simulation analysis based on the stochastic inventory model described in the previous section. The following notifications shown in Table 2 are used in this simulation model for the control of the inventory under stochastic demand.

Table2: Notification for Inventory Model

| Jnl: Cost | The costs arise from ordering the product. Unit cost includes the price was tharged ty suppliers and the procuction cơst for one unt |
| :---: | :---: |
| Holding Cost , h | The cost holding one unit $n$ stack for one period of time |
| Jacklog Cest, b(SL) | The cost of untulfilled order on a given day |
| Lead Time,LT | The time between placing and order and getting the item in stock ready for use |
| Actual Demand, Dilt | Demand in the form of bocked customer orders in given time |
| Demand Forecasting, F(t) | Estimated quantity of product that customerwill purchase in a given time |
| Order $\mathrm{Q}_{\text {dantity }} \mathrm{Q}(\mathrm{t})$ | The quantity of procmuct is ode ed so that the stock is replenished |
| Deliver Quantity, $\mathrm{R}(\mathrm{t}$ ) | The quantity of product is delivered to the stock |
| nuentory on hand, $\mathrm{J}(\mathrm{t})$ | is the current stock lovel ard the products is revdy to be distrlbuted to customer |
| Inventory Position, 1(L) | is equal to the sum of on-hand inventory and inventory on order minus backorders |
| 3ackorder, $\mathrm{B}(\mathrm{t})$ | A custemer order that cannot be filled when presented, and for which the customer is prepared to wait for some time |
| Service Level, SL | is the complement of the probability of a stockout. Service level expressed as SL= 1-(Backorder;//Actual Demand) or = 1$\mathrm{B}(\mathrm{t}) / \mathrm{D}(\mathrm{t})$ |
| wafety Stock. SS | Ihe amount of bufferis to prozect stockout event. Ihe service level led safety stock calculation. |
| Reorder level, s* | a minimum amount of an item which a company holds in stock, such that, when stock falls to this amount, the item must be reordered. |

### 5.2 Inventory process

The inventory problem on hand is to develop a model that can be used to simulate the total cost corresponding to safety stock and reorder point. The model begins each day
by checking whether any order inventories has arrived. If so the current inventory on hand must be increased by the quantity of goods received. Next the model generates a value for the weekly actual demand and forecast under the condition of $70 \%$ forecast accuracy. If there is sufficient inventory on hand to meet the demand, the stock level on hand will be decreased. If inventory on hand is not sufficient to satisfy all the demand, any unsatisfied demand will result in the backlog order, for which to compute the backlog cost. After that, if inventory on hand is lower than the base stock level, a new order should be placed as in equation 12. If an order is placed, production cost will occur. Finally, an inventory holding cost for each unit in weekly ending inventory is computed. The flow chart of the simulation for weekly operation is shown in figure 6.


Figure 6: Simulation Flowchart for a week operation Inventory Model

### 5.3 Choosing the reorder point, $s$

A common approach to choosing the reorder point s is to base it on management's desired level of service to customers. Reorder point can be calculated from equation 4.

$$
\begin{equation*}
\mathrm{s}=\mathrm{a}^{*} \mathrm{LT}+\mathrm{z}^{-1}(\mathrm{SL}) \sigma(\mathrm{LT})^{1 / 2} \tag{4}
\end{equation*}
$$

In the model, we assumed lead time and average demand per week as 16 weeks and 100 units respectively. A managerial decision needs to be made on the desired value of at least one of these measures of service level. We will denote the desired level of
service under this measure as $\mathrm{SL} . \mathrm{SL}=$ management's desired probability that a stock out event will occur in the acceptable level between the time an order quantity is placed and the order quantity is received. For example, suppose that the demand distribution is a normal distribution with some mean $\mu$ and variance $\sigma 2$ (and so standard deviation $\sigma$ ), where $\mathrm{a}=100, \sigma=140$ and $\mathrm{LT}=16$, choosing $\mathrm{SL}=0.98$ gaves

$$
\mathrm{s}=100 * 16+\mathrm{z}^{-1}(0.98) * 140 *(16)^{1 / 2}
$$

From a statistical z-table, z-value at $98 \%$ is equal to 2.05, thus reorder point is

$$
\begin{gathered}
s=100 * 16+1149 \\
=2749
\end{gathered}
$$

As the reorder point that was adopted, this provided safety stock as:

$$
\begin{gathered}
\mathrm{SS}=100 * 16-2749 \\
=1149
\end{gathered}
$$



Figure 7: The reorder point and probability distribution demand over lead time

### 5.4 The simulation result.

Each work operates on a different service level. Since the model is simulated, provided information immediately calculates the reorder point s and displays these results in the table 3. The table shows the different service level, provides the different backlog cost, ordering cost and holding cost. Total inventory cost is calculated as equation 9 . The choice of desired service level has a substantial effect on the reorder point $R$, and so on the amount of safety stock carried in inventory. Thus, the service level that gives us the minimum total inventory cost will be selected. Once known the desired service level, optimal safety stock and optimal reorder point can be defined.

Table 3: The result of simulation express in total inventory cost

| Service Level \& Cost/week Table |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| Service Level | Backlog Cost | Ordering Cost | Holding Cost | Total Cost |
| 0.5 | 277,825 | 11,077 | 5,296 | 294,198 |
| 0.6 | 193,630 | 11,077 | 6,698 | 211,405 |
| 0.7 | 112,572 | 9,231 | 9,260 | 131,063 |
| 0.75 | 67,572 | 9,231 | 11,492 | 88,295 |
| 0.8 | 34,615 | 9,231 | 13,448 | 57,294 |
| 0.85 | 26,980 | 9,231 | 13,993 | 50,204 |
| 0.9 | 19,877 | 9,231 | 16,563 | 45,671 |
| 0.95 | 13,799 | 9,231 | 19,862 | 42,892 |
| 0.99 | 5,390 | 9,231 | 30,453 | 45,074 |

The result from table 3 can be plotted in a graph as shown in figure 8 .


Figure 8: The relation between service level and total inventory cost

As shown in the graph, we may conclude that at a service level of $95 \%$ provided us with the minimum inventory cost. From equation 4, at the service level of $95 \%$, reorder point falls at 2,521 units. Therefore, if the inventory position is below the 2,521 units, an amount is ordered which equals 1,600 . The order arrives to replenish the inventory after 16 weeks.

## 6. CONCLUSION

Supply chain management is one of the key concepts that have emerged in this global economy. This concept pushes the management of an inventory system one step further. For some time, the inventory has been pushed to be lean as the just-in-time philosophy. This philosophy has enabled the company to greatly reduce its work-in-process inventories while also improving the efficiency of its production processes. Although it has been necessary to maintain some inventories of finished products until they could be sold. A very important consequence of uncertain demand is the great risk of incurring shortages unless the inventory is managed carefully. Especially, in semiconductor industry, it is evident that uncertainty in demand is highly severe according to Bullwhip Effect. This effect causes the low forecast accuracy. A better forecast can save company money. However in the situation when improving forecast accuracy is a tough task like for
semiconductor companies, they also need extremely flexible supply chains.

In this paper, we perform the flexible supply chain in the term of inventory control policy. In order to address the aspect of having low forecast accuracy, in our analysis, the actual demand and the forecasting data are generated under the low forecast accuracy's scenarios based on the realistic business situation. Moreover, the variability in demand is both driving up the average inventory level and causing significant backlog costs. Thus, another concerned issue is the backlog costs in the semiconductor industry. Estimating backlog costs requires a managerial assessment of the seriousness of making customers wait to have their orders filled. In this study, backlog cost based on EBIT is considered as an exponential function of service level. From figure 4 , it can be noticed that in a highly competitive market like the semiconductor industry, once a company cannot keep the service level at a high percentage level, they would be penalized by a large amount of backlog cost. To avoid these inventory levels running very high; it is crucial to analyze how high the inventory level should be. Therefore, the simulation model is proposed by considering the single product and ( $s, Q$ ) operational strategy to achieve high level trade-off between service level and total inventory cost. The purpose is to define the optimal service level which provides us a minimum total inventory cost. Once optimal service level is identified, it is possible to define the needed safety stock and reorder point. Understanding the relationship between individual products forecast accuracy and backlog cost function is an intriguing research direction.

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